ON THE PERIODICITY OF THE SOLAR SPOTS.

By Daniel Kirkwood.

§ *I.—The Results of Observation.*

(1.) The most ancient observations of sun-spots, of which we have any record, are those of the Chinese in the year 321, A. D. The first notice of their detection by Europeans is found in the annals of the Frankish kings. A black spot, according to Adelmus, was seen on the sun's disk, March 17th, 807, and continued visible 8 days. Similar phenomena were again observed from the 28th of May to the 26th of August, A. D. 840. The year 1096 was also signalised by the appearance of spots so large as to be visible to the naked eye. The next date, in chronological order, is that of 1161, when a spot was seen by Averroës. Finally, on the 7th, 8th, and 16th of December, 1590, "a great blacke spot on the sunne," apparently "about the bignesse of a shilling," was observed at sea by those on board the ship "Richard of Arundell."* The foregoing are, we believe, the only undoubted instances in which these phenomena were observed previous to the invention of the telescope.

(2.) From 1610 to 1750 the sun was frequently observed through instruments of various optical power, and the sparseness, or even the entire absence of spots, during considerable intervals of time, as well as their great number and magnitude at other epochs, were noticed by different astronomers. From the latter date till the close of the first quarter of the present century the solar observations were more frequent and regular; still, no idea of the prevalence of *law* in the varying numbers and magnitudes of these mysterious objects had been even conjectured. We come now, however, to a most interesting and remarkable epoch in the history of solar physics.

(3.) *The* 11–*Year Period of Schwabe.*—In 1826, Hofrath Schwabe, of Dessau, commenced a series of sun-spot observations, which have been continued without interruption to the present time (1869). On each clear day he notes the number of visible groups, giving to each a special designation, to guard against counting it twice in a single rotation of the sun. In the first year, 1826, 118 spots were observed; the number was considerably greater in 1827; and in 1828 it had increased to 225. During the next five years there was a gradual *decrease;* the minimum being reached in 1833. The results of 43 years' observations are presented at one view in the following table:

* For authorities see Humboldt's Cosmos, Vol. IV., and Chambers' Descrip. Astronomy, p. 21.

TABLE I.

Schwabe's Observations of Solar Spots.

Year.	*Days of Obs.*	*Days of no Spots.*	*New Groups.*	*Max. and Min. according to Wolf.*
1826	277	22	118	
1827	273	2	161	
1828	282	0	225	Max. (1829.5)
1829	244	0	199	
1830	217	1	190	
1831	239	3	149	
1832	270	49	84	
1833	247	139	33	Min. (1833.8)
1834	273	120	51	
1835	244	18	173	
1836	200	0	272	
1837	168	0	333	Max. (1837.2)
1838	202	0	282	
1839	205	0	162	
1840	263	3	152	
1841	283	15	102	
1842	307	64	68	
1843	312	149	34	
1844	321	111	52	Min. (1844.0)
1845	332	29	114	
1846	314	1	157	
1847	276	0	257	
1848	278	0	330	Max. (1848.6)
1849	285	0	238	
1850	308	2	186	
1851	308	0	151	
1852	337	2	125	
1853	299	3	91	
1854	334	65	67	
1855	313	146	79	
1856	321	193	34	Min. (1856.2)
1857	324	52	98	
1858	335	0	188	
1859	343	0	205	
1860	332	0	211	Max. (1860.5)
1861	322	0	204	
1862	317	3	160	
1863	330	2	124	
1864	325	4	130	
1865	307	25	93	
1866	349	76	45	
1867	312	195	25	Min. (1867.0)
1868				

(4.) This table presents a very marked periodicity; the interval between two consecutive maxima or minima, being, according to Schwabe, about 10 years. Soon after the announcement of this interesting discovery Dr. Lamont, of Munich, detected a corresponding decennial period in the variation of the magnetic needle; the epochs of maxima and minima in the latter coinciding with those in the former. These results have also been confirmed by other observers in places quite remote from each other; so that the decennial *magnetic* cycle may be regarded as well established. The equality of this period with that of the solar spots naturally suggested the hypothesis of their intimate relationship. Such a causal connection may be difficult of explanation: the fact, however, is placed beyond doubt by the researches of Wolf and Sabine.* The former, besides carefully observing the sun-spots since

* These magnetic variations, which will not be discussed in the present paper, are mentioned to give completeness of view to the phenomena under consideration. It is also worthy of remark that the Aurora Borealis is believed to exhibit a corresponding periodicity.

1847, has discussed all accessible recorded observations, both solar and magnetic, bearing on the subject. He has thus ascertained a number of epochs of maxima and minima anterior to those observed by Schwabe,—from all of which he has determined the period of the spots to be 11.11 years. He undertakes to show, moreover, that this period coincides more exactly with that of the magnetic variation than the 10–year cycle of Lamont.

(5.) *The* 56–*Year Period.*—Besides Schwabe's period of 11 years, Wolf finds a larger cycle of 55 years, in which the solar activity passes through a series of changes. It is not, however, so distinctly marked as the cycle of Schwabe. Its last maximum was about 1837, and that preceding, about 1780. The *relative* number of spots in different years, from 1749 to 1826, when Schwabe commenced his systematic observations, are given (according to Wolf) in Table II.

TABLE II.

SOLAR SPOTS, FROM 1749 TO 1825.

Year.	*Rel. no. of Spots.*	*Max.*	*Min.*	*Year.*	*Rel. no. of Spots.*	*Max.*	*Min.*
1749	63.8			1788	90.6	1788.5	
1750	68.2	1750.0		1789	85.4(?)		
1751	40.9			1790	75.2		
1752	33.2			4791	46.1		
1753	23.1(?)			1792	52.7(?)		
1754	73.8			1793	20.7(?)		
1755	6.0		1755.7	1794	23.9		
1756	8.8			1795	16.5		
1757	30.4			1796	9.4		
1758	38.3(?)			1797	5.6		
1759	48.6(?)			1798	2.8		1798.5
1760	48.9			1799	5.9		
1761	75.0	1761.5		1800	16.1		
1762	50.6			1801	30.9(?)		
1763	37.4			1802	38.3(?)		
1764	34.5			1803	50.0(?)		
1765	23.0			1804	70.0(?)	1804.0	
1766	17.5(?)		1766.5	1805	50.0(?)		
1767	33.6			1806	30.0(?)		
1768	52.2			1807	10.0(?)		
1769	85.7			1808	2.2		
1770	79.4	1770.0		1809	0.8		
1771	73.2			1810	0.0		1810.5
1772	49.2			1811	0.9		
1773	39.8			1812	5.4		
1774	47.6(?)			1813	73.7		
1775	27.5		1775.8	1814	20.0(?)		
1776	35.2(?)			1815	35.0(?)		
1777	63.0			1816	45.5	1816.8	
1778	94.8			1817	43.5		
1779	99.2	1779.5		1818	34.1		
1780	72.6(?)			1819	22.5		
1781	67.7			1820	8.9		
1782	33.2(?)			1821	4.3		
1783	22.5(?)			1822	2.9		
1784	4.4(?)		1784.8	1823	1.3		1823.2
1785	18.3			1824	6.7		
1786	60.8			1825	17.4		
1787	92.8						

(6.) *The* 233–*Day Period.*—Prof. Wolf, after carefully discussing his own and Schwabe's observations, claims to have discovered two or three minor periods of solar activity. "By projecting all the results in a continuous curve, he finds in it a series of small undulations succeeding each other at an average interval of 7.65 months,"* or 233 days.

(7.) *The 27-Day Period.*—The same astronomer thinks he has detected a short period of variation corresponding to the sun's time of rotation with respect to the earth, or about 27 days.

(8.) *The 584-Day Period.*—De La Rue, Stewart and Lœwy, have found a period varying between 18 and 20 months; the mean being about 584 days.† Other periods of maxima and minima will probably be detected; but those we have enumerated are perhaps the only ones sufficiently well established to justify any attempt at explanation.

§ *II.—Discussion of the Phenomena.*

(9.) That the solar spots are produced in some way by the planetary disturbance of the photosphere, is now generally admitted. As yet, however, the manner in which this influence is exerted, can be little more than matter of conjecture. If the action is analogous to that of the moon on the earth, the relative disturbing power of the different members of the system will be as follows:

TABLE III.

Relative Influence of the Planets on the Sun's Surface.

Name.	*Mass.*		*In Aph.*	*At M. Dist.*	*In Perih.*
Mercury	$\frac{1}{4865751}$	(Encke)	63	111	219
	$\frac{1}{3000000}$	(Leverrier)	102	180	355
Venus	$\frac{1}{401211}$		203	207	211
Earth	$\frac{1}{314760}$		95	100	105
Mars	$\frac{1}{2994790}$		2	3	4
Jupiter	$\frac{1}{1047}$		194	214	236
Saturn	$\frac{1}{3496}$		8	10	12
Uranus	$\frac{1}{24899}$		0	0	0
Neptune	$\frac{1}{18780}$		0	0	0

This table is derived from the formula

$$\delta = \frac{m}{a^3},$$

where δ represents the disturbing power of a planet, m, its mass, and a, its distance.

(10.) The connection between the number of sun-spots and the positions of the planets was noticed by Wolf as long since as 1858. In the *Comptes Rendus*, for January, 1859, he published a formula in which the number of sun-spots was made to depend on the different configurations of Venus, the Earth, Jupiter and Saturn. In the learned and interesting memoir—previously referred to—of De La Rue, Stewart and Lœwy, the causal connection between the positions of Venus and Jupiter and the behaviour of sun-spots seems to be clearly established. Professor Wil-

* Sir John Herschel, Quart. Jour. Sci., Vol. I., p. 228, April, 1864.

† Am. Journ. of Sci. and Arts, for March, 1867.

liam A. Norton, of Yale College, in his "Treatise on Astronomy," pp. 434—436, presents a brief but valuable discussion of the same subject. An inspection, however, of Table III., shows that writers generally have given undue weight to Saturn's influence. Again, although Mercury's action at aphelion is but feeble, and even at his mean distance, less than that of Venus or Jupiter, his perturbing power at *perihelion* is the greatest of all planets—a fact which certainly demands consideration in any theory which refers the origin of solar spots to planetary agency. In short, after giving the subject much study and attention, I deem it impossible, with the numbers given in table III., *and without the introduction of any modifying cause*, to establish a general correspondence between the different sun-spot periods and those of regularly recurring planetary configurations.

(11.) But the hypothesis that a particular portion of the sun's surface is more favorable to spot formation—or, in other words, more susceptible to planetary influence—than others, will, it is believed, obviate all difficulty. Is there, then, any independent probability of the truth of this hypothesis? It is well known that the formation of spots occurs chiefly between particular parallels of *latitude*, and that the numbers are greater in the northern than in the southern hemisphere. It seems, therefore, at least not improbable that a like difference may exist in regard to *longitude*. "Sömmering directs attention to the fact, that there are certain meridian belts on the sun's disk, in which he had never observed a solar-spot for many years together."* Buys-Ballot, of Utrecht, has found, from an elaborate discussion of a great number of meteorological observations, that there is a short period of variation in the amount of solar heat received by our planet; the period from maximum to maximum coinciding, at least approximately, with that of the sun's rotation with respect to the earth. Sir William Herschel also believed that one side of the sun, on account of some peculiarity in its physical constitution, was less adapted to radiate light and heat than the other.

(12.) On the hypothesis which we have ventured to suggest, the sun-spot period would be equal to the interval between two conjunctions of the disturbing planets on the heliographic meridian (designated by M) of that part of the surface most susceptible to their influence. It would depend, therefore, on the ratio of the sun's period of rotation to the interval between two consecutive conjunctions of such planets. Or, as Mercury's influence is extremely variable, a maximum would be produced by this planet's perihelion passage, when the most susceptible part of the sun's surface had the same, or nearly the same, heliocentric longitude. In order, then, to test this hypothesis, we must first inquire what is the most probable period of the sun's rotation?

(13.) On account of the *proper* motion of the solar spots, the time of the sun's rotation as determined by their *apparent* motion across the disk, varies from about 25 to 29 days. The *proper* motion of the spots

* Humboldt's Cosmos, Vol. IV., p. 378.

has recently been discussed with great labor and ability by Professor Spöerer, of Anclam, and Mr. Carrington, of England, who have shown conclusively that the rapidity of movement varies regularly with the latitude. The equatorial portions have the greatest angular velocity; in other words, the proper motion of the spots is in a direction contrary to that of the sun's rotation. The formula by which the astronomers named express the law for the dependence of the sun's apparent period of rotation on the latitude are as follows:

According to Carrington, $\xi = 865' - 165' \sin \frac{7}{4} l$ . . . (1)

" " Spöerer, $\xi = 16.°8475 - 3°.3812 \sin (41°13' + l,)$. (2)

where ξ is the arc described in a solar day. The true time of rotation is supposed to be that indicated by an equatorial spot; and on this assumption, (1) gives

$$P = 24.^{d}9711 = 24^{d}\ 23^{h}\ 18^{m}\ 23^{s} \qquad (3)$$

or, (2) gives

$$P = 24.^{d}62447 = 24^{d}\ 14^{h}\ 59^{m}\ 0^{s} \qquad (4)$$

The true value is probably between the results here given.

(14.) But will this modifying element in the theory of planetary action afford a satisfactory explanation of the periodic recurrence of maxima and minima of solar spots? Let us consider.

(*a.*) *The* 11–*Year Cycle.*—The anomalistic period of Mercury is 87.^{d}9702, and

$$87.^{d}9702 \times 46 = 4046.^{d}6292 = 11.^{y}077 = T_1 \qquad (5)$$

This is very nearly equal to Wolf's value of the cycle, and agrees at least equally well with recorded facts.* Again,

$$\frac{T_1}{163} = 24.^{d}82594 = 24^{d}\ 19^{h}\ 49^{m}\ 21^{s} \qquad (6)$$

which is nearly a mean between Spöerer's and Carrington's values of the sun's period of rotation. With this, therefore, as the time of the sun's axial revolution, we have 46 times the period of Mercury—equal to 163 times that of the sun's rotation. The recurrence of maxima at mean intervals of 11.077 years would thus be accounted for.† Again, the epochs at which sun-spots were seen before the invention of the telescope may be presumed, with much probability, to have been nearly co-incident with the maxima epochs of Schwabe's cycle. Now, it is a remarkable

* The following astronomical cycles are also nearly equal to this period of variation:

1. 18 periods of Venus = 11.074y.	4. $17t_1 = 11.030y$
2. 35 syn. per. of Mer. = 11.104	5. $28t_2 = 11.082$
3. 1 period of Jupiter. = 11.860	6. $45t_3 = 11.063$,

where t_1 = the syn. per. of Venus with respect to Jupiter; t_2 = syn. per. of Mercury with respect to Venus; and t_3 = that of Mercury with respect to Jupiter.

† It is not probable that Mercury is on the meridian M precisely at the epoch of perihelion passage. It is only necessary to suppose this coincidence to occur when the planet is *near* the perihelion point. Even at the distance of 20° the diminution of the disturbing power would be extremely small.

fact that all of those dates given in Art. (1), except perhaps the last, harmonise with the value which we have adopted for Schwabe's period of variation. Thus,

From 321, A.D.	to 1860,	we have	139	periods of	11.072+	years each.	
" 321	to 807	"	44	"	11.045	"	
" 807.22	to 840.5	"	3	"	11.093	"	
" 840.5	to 1096	"	23	"	11.109	"	
" 1096	to 1161	"	6	"	10.833	"	
" 1161	to 1590.9	"	39	"	11.024	"	
" 1590.9	to 1750.0	"	14	"	11.367(?)	"	
" 1750.0	to 1829.0	"	7	"	11.286	"	
" 1829.0	to 1860.5	"	3	"	10.500	"	

The variability of the period will be hereafter considered.

(*b.*) *Wolf's Cycle of* 56–57 *Years.*—The synodic revolution of Mercury is 115^d87748, and

$$115.^d 87748 \times 177 = 20510.^d 31396 = 56.^y 15324 = T_2 \quad . \quad . \quad (7)$$

In this period the line of conjunction of Mercury and the earth advances 56.15324 revolutions. Now,

$$\frac{T_2}{826.15324} = 24.^d 82628 = 24^d\ 19^h\ 49^m\ 50^s \quad . \quad . \quad . \quad . \quad (8)$$

This value of the sun's period of rotation differs from that in (6) by only 29 seconds. Adopting it, therefore, we find that Mercury and the earth will be in conjunction on the same heliographic meridian at regularly recurring epochs of 56 years and 56 days.

(*c.*) *The* 233–*Day Period.*—The mean interval between the consecutive conjunctions of Venus and Jupiter is 236^d992. The close agreement of these periods, leaves little room to doubt that the latter is the true period of spot variation.

(*d.*) *The* 27–*Day Period.*—This is at once satisfactorily accounted for on the hypothesis prepared in Art (11).

(*e.*) *The* 584–*Day Period.*—The identity of this period with that of the synodic revolution of Venus has already been indicated by De La Rue, Stewart and Lœwy.

(15.) It would be easy to point out theoretically other periods of variation, which an exact discussion of observations would probably confirm. It will be obvious, however, that the actual phenomena must be exceedingly complicated. The great eccentricity of Mercury's orbit; the ever-varying configurations of the disturbing planets; the probably unequal susceptibility of different parts of the sun's surface to their influence; combined, perhaps, with other causes, but imperfectly understood, must render the complete discussion of the phenomena both operose and difficult. The subject, in short, presents a new and interesting department of the theory of perturbations.

(16.) A careful inspection of tables I. and II. will indicate that Schwabe's cycle is liable to considerable variation, both in duration and intensity. The epochs of greatest disturbance were 1837 and 1848, when the number of spots was about 50 per cent. greater than in 1828 and 1860.

The observations of recent years seem to render it probable, moreover, that the epoch of extraordinary activity is passing away. The number of new groups in 1867 was less than in any other year since the commencement of Schwabe's observations; while the whole number counted during the 11 years from 1857 to 1867 inclusive, as compared with that of the 11 years immediately preceding, was as 1483 to 1715.

(17.) *The Great Irregularity of the* 11–*Year Cycle from* 1828 *to* 1860.—Mercury was in perihelion about 1838.277, and this was probably the maximum epoch *depending on Mercury alone.* But the *observed* epoch of greatest disturbance was about 1837.2. Let us, then, inquire whether any configuration of the disturbing planets will account for this marked deviation from regularity.

Mercury and Venus had the same mean longitude (343°) near the 1st of April, 1837, or, about 1837.247*, when Mercury was at less than its mean distance from the sun. If this conjunction occurred on, or very near, the solar meridian M, an extraordinary disturbance of the photosphere would evidently result. Now, the interval from 1837.247 to 1838.277 was $376.^{d}2075$, during which time the sun would have performed 15 *entire* rotations; also the arc between longitude 343° and 75°, (that of Mercury's perihelion,) is 92°. The daily motion of Mercury, moreover, when nearest the sun is about 5°. If, then, the conjunction of 1837.247 occurred over the solar meridian M, and if we represent by t the number of days from 1838.277 till Mercury was on the same solar meridian, we shall have, taking the sun's period of rotation as adopted in (6)

$$\frac{376.2075 + t}{15 + \dfrac{92 \times 5\,t}{360}} = 24.826 \qquad (9)$$

whence $t = 3.^{d}8 +$, and $5° \times t = 19° +$ . . . . . . (10)

Hence the longitude of Mercury when on the solar meridian M in 1838, and at other recent maximum epochs, was 94°, or 19° from the perihelion. Again, the interval between two consecutive conjunctions of Mercury and Venus is $144^{d}.5651$, and

$$144.^{d}5651 \times 28 = 4047.^{d}8228,$$

exceeding the period of 163 solar rotations by $1.^{d}19$. It is easy to see, therefore, that when the mean longitudes of the planets were the same (about 348°) in 1848.328, the ecliptical longitude of the solar meridian M was 12° in advance, and that the disturbing effect would consequently be diminished, although still sufficient to fix the maximum in 1848 instead of 1849. In like manner the further decrease of solar activity in 1859—60, as well as the observed increase from 1828 to 1837, is readily accounted for.

(18.) Mercury and the earth had the same mean longitude, 0°, ± about 1837.726.

Mercury and Venus, " " 343° ± " 1837.247.

* This is not the precise epoch of conjunction; we may adopt it, however, without material error. It may be remarked that a great disturbance of the photosphere would also be produced by the passage of the planets *successively* over the meridian M, shortly *before* the time of actual conjunction.

The interval between these epochs was $174.^{d}95$. Hence if these conjunctions occurred on the solar meridian M, the sun, during the interval, must have performed 7.047 revolutions. Now,

$$\frac{174.^{d}95}{7.047} = 24.^{d}826,$$

the same value of the sun's period of rotation as was found in (6). The harmony of these results affords a striking confirmation of the proposed hypothesis.

(19.) We have given a very imperfect discussion of the spot-cycles due to the disturbing effect of Mercury, Venus, and the earth. These results must be materially modified by Jupiter, whose disturbing influence has not yet been considered. It is not too much to hope that by means of a more exact analysis, in which the action of each of the planets, Mercury, Venus, the earth, and Jupiter shall be taken into account, the condition of the sun's surface may be predicted with as much certainty as the ebbing and flowing of the tides at any particular locality on the surface of our planet.

(20.) An easy calculation will show that the greatest tide produced in the sun's photosphere by any single planet must be less than an inch in height. The actual disturbance, therefore, is certainly much greater than might reasonably have been expected from a cause apparently so insignificant. It is conceivable, however, that the physical constitution of the fluids forming the luminous surface may be such that a very slight impulse may be sufficient to create a rupture, and thus occasion the phenomena observed.

(21.) The foregoing discussion justifies, we think, the following conclusions :

1. A connection between the behaviour of sun-spots and the configurations of certain planets has been placed beyond reasonable doubt.

2. The theory, however, of spot formation by planetary influence is encumbered with anomalies and even inconsistencies, unless we admit the co-operation of a modifying cause.

3. The hypothesis that a particular part of the solar surface is more susceptible than others to planetary disturbance is rendered probable by the observations of different astronomers.

4. The 11-year cycle of spot-variation is mainly dependent on the influence of Mercury.

5. The marked irregularity of this period from 1822 to 1867, is in a great measure due to the disturbing action of Venus.

6. Wolf's 56-year cycle is determined by the joint action of Mercury and the earth. And,

Finally, *the hypothesis proposed accounts, as we have seen, for all the well defined cycles of spot-variations.*

Bloomington, Indiana, *March* 15*th*, 1869.